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# SCALAR POTENTIAL IN $F(\mathcal{R})$ SUPERGRAVITY <sup>1</sup>

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## Abstract

We derive a scalar potential in the recently proposed N=1 supersymmetric generalization of  $f(\mathcal{R})$  gravity in four space-time dimensions. Any such higher-derivative supergravity is classically equivalent to the standard N=1 supergravity coupled to a chiral (matter) superfield, via a Legendre-Weyl transform in superspace. The Kaehler potential, the superpotential and the scalar potential of that theory are all governed by a single holomorphic function. We also find the conditions for the vanishing cosmological constant and spontaneous supersymmetry breaking, without fine-tuning, which define a no-scale  $F(\mathcal{R})$  supergravity. The  $F(\mathcal{R})$  supergravities are suitable for physical applications in the inflationary cosmology based on supergravity and superstrings.

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# 1 Introduction

Superstring theory naturally includes gauge field theories and gravity, it offers a consistent perturbation theory for quantum gravity, and it has promise of unifying the Standard Model of elementary particles with gravity. An experimental test of string theory may be possible by future precision data from cosmological observations. However, it requires, in the first place, a reliable theoretical derivation of inflation from string theory [1, 2, 3].

String theory needs local supersymmetry for its consistency, whereas the most direct connection to the observational cosmology is provided by the effective  $N=1$  supergravity in four space-time dimensions, after a flux compactification and moduli stabilization of the ten-dimensional superstring theory (or M-theory in eleven dimensions) [4].

In our recent paper [5] we proposed the geometrical origin of an inflaton and quintessence, as described by a dynamically generated complex scalar field in a certain higher-derivative  $N=1$  supergravity theory. Our modified supergravity [5] can be seen as the  $N=1$  locally supersymmetric extension of the  $f(R)$  gravity theories that attracted considerable interest in recent years due to their natural ability to describe the current universe acceleration and unify it with the early universe inflation [6].

The supersymmetric extension of the  $f(R)$  gravity theories we proposed in ref. [5] is non-trivial because, despite of the apparent presence of the higher derivatives of arbitrary order in the Lagrangian, there are no ghosts, potential instabilities can be avoided, and the auxiliary freedom [7] is preserved. Perhaps, most importantly, the modified supergravity action [5] is classically equivalent to the *standard*  $N=1$  supergravity minimally coupled to a chiral ‘matter’ superfield whose Kähler potential and superpotential are dictated by a single holomorphic function (see Secs. 2 and 3 below).

We speculated in ref. [5] that our  $F(\mathcal{R})$  supergravity may be dynamically generated by some non-perturbative quantum superstring corrections, whereas the complex scalar field component of the physical chiral scalar superfield (in the equivalent scalar-tensor supergravity) may be identified with a dilaton-axion pair.

In this Letter we would like to investigate some model-independent phenomenological prospects of our proposal [5] by deriving a scalar potential in our modified supergravity. We get the conditions for the vanishing cosmological constant together with spontaneous supersymmetry breaking, without fine tuning. Those no-scale supergravities are the starting point of almost any derivation of inflation from supergravity and string theory — see e.g., ref. [2] and references therein.

In Sec. 2 we give our notation and setup. In Sec.3 we recall (and generalize) the derivation [5] of the classical equivalence via a Legendre-Weyl transform in curved superspace, and add a discussion of the super-Kähler-Weyl invariance. In

Sec. 4 we introduce the no-scale modified supergravity. Sec. 4 is our conclusion.

## 2 Notation and setup

A concise and manifestly supersymmetric description of supergravity is given by superspace [8]. In this section we provide a few equations for notational purposes only.<sup>2</sup>

The chiral superspace density (in the supersymmetric gauge-fixed form) is

$$\mathcal{E}(x, \theta) = e(x) [1 - 2i\theta\sigma_a\bar{\psi}^a(x) + \theta^2 B(x)] , \quad (2.1)$$

where  $e = \sqrt{-\det g_{\mu\nu}}$ ,  $g_{\mu\nu}$  is a spacetime metric,  $\psi_\alpha^a = e_\mu^a \psi_\alpha^\mu$  is a chiral gravitino,  $B = S - iP$  is the complex scalar auxiliary field. We use the lower case middle greek letters  $\mu, \nu, \dots = 0, 1, 2, 3$  for curved spacetime vector indices, the lower case early latin letters  $a, b, \dots = 0, 1, 2, 3$  for flat (target) space vector indices, and the lower case early greek letters  $\alpha, \beta, \dots = 1, 2$  for chiral spinor indices.

The solution of the superspace Bianchi identities and the constraints defining the  $N = 1$  Poincaré-type minimal supergravity results in only three relevant superfields  $\mathcal{R}$ ,  $\mathcal{G}_a$  and  $\mathcal{W}_{\alpha\beta\gamma}$  (as parts of the supertorsion), subject to the off-shell relations [8]

$$\mathcal{G}_a = \bar{\mathcal{G}}_a , \quad \mathcal{W}_{\alpha\beta\gamma} = \mathcal{W}_{(\alpha\beta\gamma)} , \quad \bar{\nabla}_{\dot{\alpha}} \mathcal{R} = \bar{\nabla}_{\dot{\alpha}} \mathcal{W}_{\alpha\beta\gamma} = 0 , \quad (2.2)$$

and

$$\bar{\nabla}_{\dot{\alpha}} \mathcal{G}_{\alpha\dot{\alpha}} = \nabla_{\alpha} \mathcal{R} , \quad \nabla^{\gamma} \mathcal{W}_{\alpha\beta\gamma} = \frac{i}{2} \nabla_{\alpha} \mathcal{G}_{\beta\dot{\alpha}} + \frac{i}{2} \nabla_{\beta} \mathcal{G}_{\alpha\dot{\alpha}} , \quad (2.3)$$

where  $(\nabla_{\alpha}, \bar{\nabla}_{\dot{\alpha}}, \nabla_{\alpha\dot{\alpha}})$  represent the curved superspace  $N = 1$  supercovariant derivatives, and bars denote complex conjugation.

The covariantly chiral complex scalar superfield  $\mathcal{R}$  has the scalar curvature  $R$  as the coefficient at its  $\theta^2$  term, the real vector superfield  $\mathcal{G}_{\alpha\dot{\alpha}}$  has the traceless Ricci tensor,  $R_{\mu\nu} + R_{\nu\mu} - \frac{1}{2}g_{\mu\nu}R$ , as the coefficient at its  $\theta\sigma^a\bar{\theta}$  term, whereas the covariantly chiral, complex, totally symmetric, fermionic superfield  $\mathcal{W}_{\alpha\beta\gamma}$  has the Weyl tensor  $W_{\alpha\beta\gamma\delta}$  as the coefficient at its linear  $\theta^\delta$ -dependent term.

A generic supergravity Lagrangian (e.g. representing the supergravitational part of the superstring effective action) is

$$\mathcal{L} = \mathcal{L}(\mathcal{R}, \mathcal{G}, \mathcal{W}, \dots) \quad (2.4)$$

where the dots stand for arbitrary covariant derivatives of the supergravity superfields. We would like to concentrate on the particular sector of the theory (2.4), by ignoring the tensor superfields  $\mathcal{W}_{\alpha\beta\gamma}$  and  $\mathcal{G}_{\alpha\dot{\alpha}}$ , and the derivatives of the

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<sup>2</sup>We use the natural units  $c = \hbar = \kappa = 1$ .

scalar superfield  $\mathcal{R}$ . Thus the effective higher-derivative supergravity action we proposed [5] is given by

$$S_F = \int d^4x d^2\theta \mathcal{E} F(\mathcal{R}) + \text{H.c.} \quad (2.5)$$

with some holomorphic function  $F(\mathcal{R})$  presumably generated by strings. Besides manifest local  $N = 1$  supersymmetry, the action (2.5) also possesses the auxiliary freedom [7], since the auxiliary field  $B$  does not propagate. It distinguishes the action (2.5) from other possible truncations of eq. (2.4). In addition, the action (2.5) gives rise to a spacetime torsion.

### 3 Super-Legendre-Weyl-Kähler transform

The superfield action (2.5) is classically equivalent to another action

$$S_V = \int d^4x d^2\theta \mathcal{E} [\mathcal{Z}\mathcal{R} - V(\mathcal{Z})] + \text{H.c.} \quad (3.6)$$

where we have introduced the covariantly chiral superfield  $\mathcal{Z}$  as a Lagrange multiplier. Varying the action (3.6) with respect to  $\mathcal{Z}$  gives back the original action (2.5) provided that

$$F(\mathcal{R}) = \mathcal{R}\mathcal{Z}(\mathcal{R}) - V(\mathcal{Z}(\mathcal{R})) \quad (3.7)$$

where the function  $\mathcal{Z}(\mathcal{R})$  is defined by inverting the function

$$\mathcal{R} = V'(\mathcal{Z}) \quad (3.8)$$

Equations (3.7) and (3.8) define the superfield Legendre transform. They imply further relations,

$$F'(\mathcal{R}) = \mathcal{Z}(\mathcal{R}) \quad \text{and} \quad F''(\mathcal{R}) = \mathcal{Z}'(\mathcal{R}) = \frac{1}{V''(\mathcal{Z}(\mathcal{R}))} \quad (3.9)$$

where  $V'' = d^2V/d\mathcal{Z}^2$ . The second formula (3.9) is the duality relation between the supergravitational function  $F$  and the chiral superpotential  $V$ .

A super-Weyl transform of the superfield action (3.6) can also be done entirely in superspace, i.e. with manifest local  $N=1$  supersymmetry. In terms of components, the super-Weyl transform amounts to a Weyl transform, a chiral rotation and a (superconformal)  $S$ -supersymmetry transformation [9].

The chiral density superfield  $\mathcal{E}$  is a chiral compensator of the super-Weyl transformations

$$\mathcal{E} \rightarrow e^{3\Phi} \mathcal{E} , \quad (3.10)$$

whose parameter  $\Phi$  is an arbitrary covariantly chiral superfield,  $\bar{\nabla}_{\dot{\alpha}} \Phi = 0$ . Under the transformation (3.10) the covariantly chiral superfield  $\mathcal{R}$  transforms as

$$\mathcal{R} \rightarrow e^{-2\Phi} \left( \mathcal{R} - \frac{1}{4} \bar{\nabla}^2 \right) e^{\Phi} \quad (3.11)$$

The super-Weyl chiral superfield parameter  $\Phi$  can be traded for the chiral Lagrange multiplier  $\mathcal{Z}$  by using a generic gauge condition <sup>3</sup>

$$\mathcal{Z} = \mathcal{Z}(\Phi) \quad (3.12)$$

where  $\mathcal{Z}(\Phi)$  is an arbitrary (holomorphic) function of  $\Phi$ . Then the super-Weyl transform of the action (3.6) results in the classically equivalent action

$$S_\Phi = \int d^4x d^4\theta E^{-1} e^{\Phi+\bar{\Phi}} [\mathcal{Z}(\Phi) + \bar{\mathcal{Z}}(\bar{\Phi})] + \left[ - \int d^4x d^2\theta \mathcal{E} e^{3\Phi} V(\mathcal{Z}(\Phi)) + \text{H.c.} \right] , \quad (3.13)$$

where we have introduced the full supergravity supervielbein  $E^{-1}$  [8].

Equation (3.13) has the standard form of the action of a chiral matter superfield coupled to supergravity [8],

$$S[\Phi, \bar{\Phi}] = \int d^4x d^4\theta E^{-1} \mathcal{O}(\Phi, \bar{\Phi}) + \left[ \int d^4x d^2\theta \mathcal{E} P(\Phi) + \text{H.c.} \right] , \quad (3.14)$$

in terms of a ‘Kähler’ potential  $\mathcal{O}(\Phi, \bar{\Phi})$  and a chiral superpotential  $P(\Phi)$ . In our case (3.13) we find

$$\begin{aligned} \mathcal{O}(\Phi, \bar{\Phi}) &= e^{\Phi+\bar{\Phi}} [\mathcal{Z}(\Phi) + \bar{\mathcal{Z}}(\bar{\Phi})] , \\ P(\Phi) &= -e^{3\Phi} V(\mathcal{Z}(\Phi)) \end{aligned} \quad (3.15)$$

The truly Kähler potential  $K(\Phi, \bar{\Phi})$  is given by [8]

$$K = -3 \ln\left(-\frac{\mathcal{O}}{3}\right) \quad \text{or} \quad \mathcal{O} = -3e^{-K/3} , \quad (3.16)$$

because of the invariance of the action (3.14) under the supersymmetric Kähler-Weyl transformations

$$K(\Phi, \bar{\Phi}) \rightarrow K(\Phi, \bar{\Phi}) + \mathbf{L}(\Phi) + \bar{\mathbf{L}}(\bar{\Phi}) , \quad \mathcal{E} \rightarrow e^{\mathbf{L}(\Phi)} \mathcal{E} , \quad P(\Phi) \rightarrow -e^{-\mathbf{L}(\Phi)} P(\Phi) \quad (3.17)$$

with an arbitrary chiral superfield parameter  $\mathbf{L}(\Phi)$ .

The scalar potential (in components) is given by the standard formula [10]

$$\mathcal{V}(\phi, \bar{\phi}) = e^{\mathcal{O}} \left\{ \left| \frac{\partial \mathcal{O}}{\partial \Phi} + \frac{\partial \mathcal{O}}{\partial \bar{\Phi}} P \right|^2 - 3 |P|^2 \right\} , \quad (3.18)$$

where all superfields are restricted to their leading field components,  $\Phi| = \phi(x)$ . Equation (3.18) can be simplified by making use of the Kähler-Weyl invariance (3.17) that allows us to choose the gauge

$$P = 1 \quad (3.19)$$

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<sup>3</sup>In ref. [5] we used the particular gauge  $\xi\Phi = \ln \mathcal{Z}$  with a number  $\xi$ .

It is equivalent to the well known fact that the scalar potential (3.18) is actually governed by the single (Kähler-Weyl invariant) potential [8]

$$G(\Phi, \bar{\Phi}) = \mathcal{O} + \ln P + \ln \bar{P} \quad (3.20)$$

In our case (3.15) we have

$$G = e^{\Phi + \bar{\Phi}} [\mathcal{Z}(\Phi) + \bar{\mathcal{Z}}(\bar{\Phi})] + 3(\Phi + \bar{\Phi}) + \ln(-V(\mathcal{Z}(\Phi))) + \ln(-\bar{V}(\bar{\mathcal{Z}}(\bar{\Phi}))) \quad (3.21)$$

Let's now specify our gauge (3.12) by choosing the condition

$$3\Phi + \ln(-V(\mathcal{Z}(\Phi))) = 0 \quad \text{or} \quad V(\mathcal{Z}(\Phi)) = -e^{-3\Phi} \quad (3.22)$$

that is equivalent to eq. (3.19). Then the potential (3.21) gets simplified to

$$G = \mathcal{O} = e^{\Phi + \bar{\Phi}} [\mathcal{Z}(\Phi) + \bar{\mathcal{Z}}(\bar{\Phi})] \quad (3.23)$$

Equations (3.7), (3.8) and (3.23) are the simple one-to-one algebraic relations between a holomorphic function  $F(\mathcal{R})$  in our modified supergravity action (2.5) and a holomorphic function  $\mathcal{Z}(\Phi)$  entering the potential (3.23) and defining the scalar potential (3.18) as

$$\mathcal{V} = e^G \left[ \left( \frac{\partial^2 G}{\partial \Phi \partial \bar{\Phi}} \right)^{-1} \frac{\partial G}{\partial \Phi} \frac{\partial G}{\partial \bar{\Phi}} - 3 \right] \quad (3.24)$$

in the classically equivalent scalar-tensor supergravity. The latter can be used for embedding the standard slow-roll inflation [11] into supergravity. In our setup that correspondence can be promoted further, by embedding the slow-roll inflation into the ‘purely geometrical’ higher-derivative supergravity theory (2.5), in terms of a single holomorphic function. For our purposes below, we can restrict our attention to eqs. (3.23) and (3.24) in terms of a function  $\mathcal{Z}(\Phi)$  only.

## 4 No-scale modified supergravity

The no-scale supergravity arises by demanding the scalar potential (3.24) to vanish. It results in the vanishing cosmological constant without fine-tuning [12]. The no-scale supergravity potential  $G$  has to obey the non-linear 2nd-order partial differential equation

$$3 \frac{\partial^2 G}{\partial \Phi \partial \bar{\Phi}} = \frac{\partial G}{\partial \Phi} \frac{\partial G}{\partial \bar{\Phi}} \quad (4.25)$$

A gravitino mass  $m_{3/2}$  is given by the vacuum expectation value [8]

$$m_{3/2} = \langle e^{G/2} \rangle \quad (4.26)$$

so that the spontaneous supersymmetry breaking scale can be chosen at will.

The well known exact solution to eq. (4.25) is given by

$$G = -3 \ln(\Phi + \bar{\Phi}) \quad (4.27)$$

In the recent literature, the no-scale solution (4.27) is usually modified by other terms, in order to achieve the universe with a positive cosmological constant — see e.g., the KKLT mechanism [13].

To appreciate the difference between the standard no-scale supergravity solution and our modified supergravity, it is worth noticing that the ansatz (4.27) is inconsistent with our potential (3.23) by any choice of the function  $\mathcal{Z}$ . Instead, in our case, demanding eq. (4.25) gives rise to the 1st-order non-linear partial differential equation

$$3 \left( e^{\bar{\Phi}} X' + e^{\Phi} \bar{X}' \right) = \left| e^{\bar{\Phi}} X' + e^{\Phi} \bar{X}' \right|^2, \quad (4.28)$$

where we have introduced the notation

$$\mathcal{Z}(\Phi) = e^{-\Phi} X(\Phi), \quad X' = \frac{dX}{d\Phi}, \quad (4.29)$$

in order to get the differential equation in its most symmetric and concise form.

Accordingly, the gravitino mass (4.26) is given by

$$m_{3/2} = \left\langle \exp \frac{1}{2} \left( e^{\bar{\Phi}} X + e^{\Phi} \bar{X} \right) \right\rangle \quad (4.30)$$

I am not aware of any non-trivial holomorphic exact solution to eq. (4.28). Should it obey a holomorphic differential equation of the form

$$X' = e^{\Phi} g(X, \Phi) \quad (4.31)$$

with a holomorphic function  $g(X, \Phi)$ , eq. (4.28) gives rise to a functional equation,

$$3(g + \bar{g}) = \left| e^{\bar{\Phi}} g + \bar{X}' \right|^2 \quad (4.32)$$

When being restricted to the real variables  $\Phi = \bar{\Phi} \equiv y$  and  $X = \bar{X} \equiv x$ , eq. (4.28) reads

$$6x' = e^y (x' + x)^2, \quad \text{where} \quad x' = \frac{dx}{dy} \quad (4.33)$$

This equation can be integrated after a change of variables,<sup>4</sup>

$$x = e^{-y} u, \quad (4.34)$$

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<sup>4</sup>I am grateful to A. Starobinsky who pointed it out to me.

which leads to a quadratic equation with respect to  $u' = du/dy$ ,

$$(u')^2 - 6u' + 6u = 0 \quad (4.35)$$

It follows

$$y = \int^u \frac{d\xi}{3 \pm \sqrt{3(3-2\xi)}} = \mp \sqrt{1 - \frac{2}{3}u} + \ln \left( \sqrt{3(3-2u)} \pm 3 \right) + C . \quad (4.36)$$

## 5 Discussion

Our main new result is given by eq. (3.23) together with the origin of a function  $\mathcal{Z}$  out of the action (2.5). It implies the existence of new no-scale supergravities based on exact solutions to eq. (4.28), with the vanishing cosmological constant and spontaneously broken supersymmetry, without fine-tuning.

A generic modified supergravity potential (3.23) in terms of the holomorphic pre-potential  $X(\Phi)$  defined by eq. (4.29),

$$G = \emptyset = e^{\bar{\Phi}} X(\Phi) + e^{\Phi} \bar{X}(\bar{\Phi}) , \quad (5.37)$$

has essentially the same structure as the supergravity potential of N=2 extended supergravity (coupled to N=2 vector matter) that is also governed by a holomorphic pre-potential. I do not know whether it is merely a coincidence or not.

As regards the non-vanishing (inflaton) scalar potentials from our modified supergravity, they have to satisfy the slow-roll conditions (cf. ref. [14])

$$\left| \frac{\partial \mathcal{V}}{\partial \phi} \right|^2 \ll \frac{\partial^2 K}{\partial \phi \partial \bar{\phi}} \mathcal{V}^2 \quad (5.38)$$

and

$$\left| \frac{\partial^2 \mathcal{V}}{\partial \phi \partial \bar{\phi}} + \frac{\partial K}{\partial \bar{\phi}} \frac{\partial \mathcal{V}}{\partial \phi} \right|^2 \ll \left( \frac{\partial^2 K}{\partial \Phi \partial \bar{\Phi}} \right)^2 \mathcal{V}^2 , \quad (5.39)$$

which guarantee that the potential is sufficiently flat (i.e. suitable for inflation).

Sometimes it can be achieved by demanding a shift symmetry of the Kähler potential (i.e. its flatness in one direction), and then perturbing the potential around the flat direction — see e.g., ref. [15]. Our Ansatz (3.23) for the supergravity potential can allow a shift symmetry, though it appears to be more restrictive.

In the context of superstring theory, the effective supergravity potential (3.23) may capture some ‘stringy’ features, such as duality and maximal curvature — see e.g., ref. [16].



## References

- [1] R.H. Brandenberger, Lecture Notes Phys. **646** (2004) 127, hep-th/0306071
- [2] R. Kallosh, Lecture Notes Phys. **738** (2008) 119, hep-th/0702059
- [3] M. Iihoshi and S.V. Ketov, Advances in High Energy Physics (2008) 521389, arXiv:0707.3359 [hep-th]
- [4] M. R. Douglas and S. Kachru, Rev. Mod. Phys. **79** (2007) 733, hep-th/0610102
- [5] S. James Gates, Jr. and S. V. Ketov, Superstring-inspired supergravity as the universal source of inflation and quintessence, arXiv:0901.2467[hep-th]
- [6] S. Nojiri and S.D. Odintsov, Int. J. Geom. Meth. Mod. Phys. **4** (2007) 115, hep-th/0601213;  
T.P. Sotiriou, V. Faraoni, *f(R) theories of gravity*, arXiv:0805.1726[hep-th]
- [7] S. J. Gates, Jr., Phys. Lett. **B365** (1996) 132 [hep-th/9508153], and Nucl. Phys. **B485** (1997) 145 [hep-th/9606109]
- [8] S. J. Gates, Jr., M. T. Grisaru, M. Roček and W. Siegel, *Superspace or 1001 Lessons in Supersymmetry*, Benjamin-Cummings Publ. Company, 1983;  
J. Wess and J. Bagger, *Supersymmetry and Supergravity*, Princeton University Press, 1992;  
I. L. Buchbinder and S. M. Kuzenko, *Ideas and Methods of Supersymmetry and Supergravity*, IOP Publishers, 1998
- [9] P. S. Howe and R. W. Tucker, Phys. Lett. **B80** (1978) 138
- [10] E. Cremmer, B. Julia, J. Scherk, S. Ferrara, L. Girardello and P. van Nieuwenhuizen, Nucl. Phys. **B147** (1979) 105
- [11] A. R. Liddle and D. H. Lyth, *Cosmological Inflation and Large-Scale Structure*, Cambridge University Press, 2000
- [12] E. Cremmer, S. Ferrara, C. Kounnas and D. V. Nanopoulos, Phys. Lett. **B133** (1983) 61
- [13] S. Kachru, R. Kallosh, A. Linde and S. P. Trivedi, Phys. Rev. **D68** (2003) 046005, hep-th/0301240
- [14] M. Sasaki and E. D. Stewart, Progr. Theor. Phys. **95** (1996) 71, astro-ph/9507001

- [15] M. K. Gaillard, H. Murayama and K. A. Olive, Phys. Lett. **B355** (1995) 71, hep-ph/9504307;  
M. Kawasaki, M. Yamaguchi and T. Yanagida, Phys. Rev. Lett. **85** (2000) 3572, hep-ph/0004243
- [16] S. J. Gates, Jr. and S. V. Ketov, Class. and Quantum Grav. **17** (2001) 3561, hep-th/0104223.